THE BEHAVIOR OF SHELLS UNDER IMPULSIVE LOADING. SMALL PERTURBATIONS

A. G. Ivanov, V. A. Ogorodnikov,
and E. S. Tyun'kin

UDC 539.3;534.1

The study of the behavior of thin-walled constructions and their elements (shells, rods, etc.) under the action of impulsive loads is relevant to a number of rapidly developing branches of the mechanics of deformable bodies. One of these areas of study is related to the instability of shells. A qualitative picture of the phenomenon comes down to the following. During the loading of an ideal shell by a uniformly distributed exterior impulsive load, all of its elements have the same initial radial velocity, directed toward the axis. If the stresses that arise reach the value of the dynamic yield point of the shell material, then the material begins to flow, and continues to do so until the initial kinetic energy of the shell is expended in the work of plastic deformation. A shell of smaller radius and greater thickness results. However, the presence of imperfections in the shell and nonuniformities in the load can lead to some dispersion in the values of the velocity and the finite displacements along the surface of the shell, i.e., to small perturbations in the radial compression of the material. Any deviation of the shell from a round form is increased under the action of compressive circumferential stresses. The motion which arises in this case can be accompanied, under certain conditions, by a loss of stability. If the elastic limit of the material is exceeded, a shell which loses its stability keeps its bulged form.

It is of interest to uncover the basic factors which control such motion (initial geometry and velocity of the shell, and the physicomechanical properties of the material).

1. The phenomenon of dynamic elastic instability and dynamic loss of stability beyond the limits of material elasticity for cylindrical shells under the action of exterior impulsive loading has been studied in a number of works [1-7]. Thus in [2, 3], it was established that for relatively low loading amplitudes (when the shell material is elastic: $W_0/c < \varepsilon_S$), the perturbed form of the shell, defined as the number of waves (creases) in the circumferential direction, is given by $n_1 = (2R_0/a)(W_0/c)^{1/2}$. Here W_0, c are the initial shell velocity and the sound speed, ε_S the material strain at the plastic yield point, and R_0 and a are the initial mean radius and the thickness of the shell.

According to [1], for relatively high load levels when the shell material is in the plastic state ($W_0/c \geq \varepsilon_S$), the perturbed form of a shell made of viscoplastic material is $n_2 = (\sqrt{6}R_0/a)(\sigma_T/E_h)^{1/2}$ (σ_T is the stress in the σ–ε diagram for $\varepsilon = 0.2\%$, E_h is the modulus of strain hardening).

In [4] those circumstances were considered in which the formation of waves in the circumferential direction of the shell takes place as a result of the interaction between the membrane and bending modes of deformation, which are the components of the shell's reaction to impulsive loading. For this case, [4] develops a unified approach for determining the predominant modes/form in the elastic and plastic stages of shell reaction, leading to the relation

$$n_3 = \frac{2{,}1R_0}{a}\left\{\left(\frac{\sigma_T}{E_h}\right)^2 + \left(\frac{E}{E_h}\right)\left(\frac{W_0}{c}\right)^2\right\}^{1/4}$$

(E is Young's modulus).

In [7], the motion of the boundaries of a plane viscoplastic ring was studied, in terms of the inertia towards its center, for stability of the vector velocity and the stress tensor with respect to small harmonic perturbations of the boundary. From an analysis of the asymptotes to the solution of this problem, it transpires that small perturbations on the outer boundary of the shell grow without bound during compression toward the center, while perturbations at the inner boundary have a wave-like character with limited amplitude. A relation determining the number of waves for which instability of the form is observed is written in

Arzamas-16. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 112-115, November-December, 1992. Original article submitted June 18, 1991. Revision submitted June 30, 1991.

the form $n_4 = (4\mu/R_1)(2/\rho\sigma_s)^{1/2}$ (μ is the viscosity of the shell material; R_1 the internal stopping radius of the ring; σ_s the dynamic plastic yield point).

2. In this paper, we present new experimental data on the instability of the compression of shells made of standard grade steel St3 and lead L1. In the experiments, we used cylindrical shells with a length-to-diameter ratio of ~5. The surface finish of the machined shells corresponded to class six. A layer h of plastic high explosive (HE) was loaded against the end surface of the steel shell. This high explosive had a density of 1.51 g/cm^3 and a detonation velocity of 7.8 km/sec. The shell with its HE load was enclosed in a metal shield (a shell of thickness H [8]); in a number of cases, the shield was not used, as in [1, 9]. In the experiments with lead shells, there was a uniform air gap of 2.8 mm between the loaded surface of the shell and the HE layer. In this case, the HE layer was placed on the inner surface of the shield. One of the ends of such shells was covered by a solid disk with a layer of HE applied to its surface. Detonation was initiated at the disk center (on the shell axis) and propagated out radially to the HE layer located on the side walls of the shell (or on the screen), so that loading took place under conditions of running detonation.

We established that for cylindrical shells, there is a critical value of the relative thickness ($\delta_* = a/R_{20}$; R_{20} is the outer radius of the shell), below which the shell converges unstably. Thus for the shells studied of steel and lead, the value of δ_* was ~10 and 30%, respectively.

In experiments using shells with $\delta \leq \delta_*$, for which inertial axisymmetric compression was realized when the initial kinetic energy of the shell is completely transformed during compression to the work of plastic deformation and the shell is stopped at some definite radius. This makes it possible to monitor the transverse section of the deformed shell after the experiment, and to visually observe the number of waves (creases) N in the circumferential direction. In some experiments with steel shells whose thickness was greater than the critical value, spallation of a layer of thickness $\delta_- < \delta_*$ took place [9]. In this case the main mass of the shell of thickness $\delta_+ > \delta_*$ is insignificantly displaced, with loss of stability of form, and the instability of the compression of the spalled layer was studied. In experiments with steel shells, the initial relative thickness and velocity of the shells was varied, while in experiments with lead shells, the initial velocity of the shell varied.

The conditions and results for all experiments are given in Table 1, where $\varepsilon_{2*} = R_{20} - R_{2*}/R_{20}$ is the mean relative deformation of the shell after deformation, and $\dot{\varepsilon} = W_0/R_{20}$ is the initial strain rate of the shell. For the initial shell velocity W_0, we apply its value as determined in series I from the conservation laws [10], in series II as a result of numerical computation [9], and in series III as a result of measuring the velocity of lead plates in a plane system with gaps. Here the values $n_1, \ldots, n_4$ are also introduced. In computing them, we used material properties, taking into account the dependence of these properties on strain rate [11, 12]. For steel, these are $\mu = 4.7 \cdot 10^4$ kg/(m·sec), $\sigma_T/E_h = 0.10$, $E/E_h = 82$, $\sigma_s = 1$ GPa, and for lead they are $\mu = 3.7 \cdot 10^3$ kg/(m·sec), $\sigma_T/E_h = 0.15$, $E/E_h = 100$, $\sigma_s = 30$ MPa.

TABLE 1

Material	Series	Expt. No.	Shell R_{20}, mm	Shell a, mm	Shield material	Shield H, mm	HE h, mm	W_0, m/sec	$\dot{\varepsilon} \cdot 10^4$	ε_{2*}, %	R_1, mm	N	n_1	n_2	n_3	n_4
Steel	I	1	22,5	1.87	D16	1	0,25	179	0.8	8.9	18.5	10	5	9	15	5
		2	22,2	1.82	D16	1	0,41	227	1.0	16.7	16,0	12	5	9	17	6
		3	22,5	1,88	D16	2	0,51	324	1.5	28.0	13,5	12—13	6	9	19	7
		4	22,5	1.78	St 3	2	0,67	393	1.8	42.2	4.0	16	7	9	22	24
		5	22,4	1.20	D16	1	0,49	330	1.5	44.4	10,0	12—14	10	14	30	10
Steel (spall)	II	6	20,7	2,50	—	—	3.00	400	2.0	32.0	6.0	5	5	6	14	16
		7	20,0	1,50	—	—	2.00	300	1.5	28.0	14,0	8—9	7	10	20	7
		8	20,0	0.60	—	—	0.59	167	1.0	3.4	18,0	12—14	12	25	41	5
Lead	III	9	22,0	6,00	D16	1	0.36	20	0.1	16.0	12,0	5—6 (12...15)	1	3	3	3
		10	22,0	6,00	D16	3	0.36	30	0.2	25.0	9,0	5—6 (12...15)	1	3	3	4

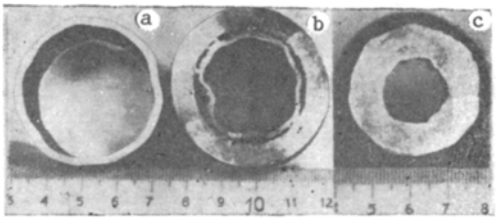

Fig. 1

Figure 1 shows a photograph of the cross sections of some shells after loading: a solid steel shell, a steel shell that has spalled, and a solid lead shell (corresponding to experiments 1, 7, and 10). Analysis of these cross sections indicates that both for solid steel shells (a) and for layers that have been spalled from solid steel shells (b), the perturbed inner and outer boundaries move in concert regardless of the initial velocity of the shell. Such behavior is not a property of the lead shells (c), which are characterized by the presence of fundamental perturbed modes/forms ($N^0 = 5$-6), on which additional perturbations are applied with approximately double frequency ($N^p = 12$-15). This is particularly clearly demonstrated for the inner shell boundary. Similar behavior was observed in experiments on axisymmetric compression of shells made of Duralumin D-16, which are compressed by high explosive products [7]. Note that the amplitudes of the perturbation both for the outer and the inner boundaries of the shells studied in [7], and the spalled layers decreases with decreasing stopping radius (or increasing loading), which is not in agreement with the asymptotic behavior in [7].

Analysis of the experimental data given in Table 1 shows that the bending deformation mode for steel shells in the plastic stage is controlled both by the magnitude of the loading (experiments 1-4) and their initial geometry (experiments 6-8). It is clear that satisfactory agreement between experimental values of the form number N with computed n_1, n_2, n_3, n_4 is on the whole not observed. For sheels being impulsively loaded by loads of various magnitudes and placed in direct contact with explosive products (experiments 1-5), the experimental values of N are confined to the range $n_2 < N < n_3$, which does not contradict the conclusions of [4]. For shells of different initial thickness (experiments 6-8), which manifest spalled layers that are not in contact with explosive products, the observed form numbers lie within other limits ($n_1 \leq N < n_2$). The results of experiment 8 ($N = n_1$, $N < n_2$, n_3) can evidently be explained by the fact that the deformation of the shell is modest ($\varepsilon_{2*} = 3.4\%$) and the elastic reaction which takes place before and after the plastic stage has a noticeable influence on the shell's behavior [4]. The observed form numbers for the lead shells do not agree with any of the computed values ($N > n_1, \ldots, n_4$). In addition, as noted above, a more complex perturbed form of the shell boundary occurs, which does not lie even qualitatively within the limits of the known asymptotes (for example, [7]).

Thus we have confirmed that with decreasing relative thickness or with growth in the initial velocity, the number of perturbations on the shell in the circumferential direction increases. The issue of how the material parameters of the shell (ρ_0, σ_S, μ) control the form of the unstable motion requires a more detailed study, especially in the area of theory. On the whole, we must recognize the validity of the assertion in [13], that at present there are no generally applicable criteria for the types of shell motion instability studied here.

LITERATURE CITED

1. G. R. Abrahamson, and J. N. Goodier, "Dynamic plastic-flow buckling of a cylindrical shell from a uniform radial impulse," in 4th U. S. National Congress of Applied Mechanics (Berkeley), Vol. 2, ASME, New York (1962).
2. G. Guder and I. Makaivor, "Elastic cylindrical shell under the action of a nearly uniform impulse," ASME Applied Mechanics, No. 2 (1964).
3. H. Lindberg, "Bending of a very thin cylindrical shell under the action of impulsive pressure," ASME Applied Mechanics, No. 2 (1964).
4. V. Stuiver, "Bulging of rings under impulsive pressure," ASME Applied Mechanics, No. 3 (1965).
5. A. Florens, "Stability loss of viscoplastic cylindrical shells during impulsive loading," RTK, No. 3 (1968).
6. A. G. Ivanov, V. N. Mineev, and E. S. Tyun'kin, "Impulsive collapse of steel cylindrical shells," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2 (1982).

7. S. V. Serikov, "The stability of a viscoplastic ring," Prikl. Mekh. Tekh. Fiz., No. 1 (1984).
8. V. A. Ogorodnikov, A. A. Sadovoi, V. N. Sofronov, and E. S. Tyun'kin, "Experimental-numerical investigation of the compression of cylindrical shells by explosions, taking dissipative processes into account," Vol. 2, Works of the 5th All-Union Conference on Detonation, Krasnoyarsk (1991).
9. A. G. Ivanov, V. N. Sofronov, and E. S. Tyun'kin, "Properties of axisymmetric compression of spalled layers in cylindrical steel shells," Prikl. Mekh. Tekh. Fiz., No. 3 (1984).
10. A. G. Ivanov, L. I. Kochkin, V. A. Ogorodnikov, and E. S. Tyun'kin, "Properties of the acceleration of plates by grazing detonation waves with combined or lumped masses," Prikl. Mekh. Tekh. Fiz., No. 5 (1990).
11. G. V. Stepanov, Elastic-Plastic Deformation of Materials Under the Action of Impulsive Loads [in Russian], Naukova Dumka, Kiev (1979).
12. M. Malatynski, and J. Klepaczko, "Experimental investigation of plastic properties of lead over a wide range of strain rates," Inter. J. Mech. Sci., 22 (1980).
13. A. V. Karmishin, A. I. Zhukov, V. G. Kolosov, et al., Methods of Dymanic Calculation and Testing of Thin-Walled Constructions [in Russian], Mashinostroenie, Moscow (1989).

NUMERICAL ANALYSIS OF CRACK DEVELOPMENT IN STRUCTURALLY NONUNIFORM COMPOSITES

E. A. Lankina

UDC 539.2

The plane problem of an elastic unidirectional composite with a crack which grows from infinity at constant stress σ is considered here. The y axis coincides with the direction of reinforcement. If the dimensions of the binder H and the fiber h are small compared with the length of the crack, the macroscopic field far from the crack can be determined by the methods of continuum mechanics on the basis of integral equations from potential theory. Solution of the macroscopic problem in this formulation gives infinite growth of the stress upon approach to the crack margins. In the neighborhood of the crack margin, a formulation that takes into account the characteristic dimension of the real structure of the materials is necessary. Therefore, it is useful to break the problem down into two stages. In the first, the stress in a structureless composite is determined, i.e., the limiting case of a "smeared" structure is studied, when h, H → 0, h/H = const. Then a region around the crack margins in selected, and the stress determined for the smeared composite is used as a boundary condition on the boundary of this region. In the second stage, the interior of this region is described by equations that take into account the discrete structure of the composite, resulting in finite stresses. In this case the crack and the boundary of the selected region are considered to be an aggregate of fiber fractures and delaminations of binder. By using the strength conditions for fracture or delamination, the development of a crack is calculated, and parameter values are found at which cracks grow by fracture of the fibers or by delamination of the binder.

1. We denote displacement of the i-th fiber along the direction of reinforcement y by $u_i(y)$. Then the equation of equilibrium for the i-th fiber inside the composite is written as [17].

$$hH\frac{d^2u_i}{dy^2} + \beta^2(u_{i+1} - 2u_i + u_{i-1}) = 0, \quad \beta^2 = \mu/E \tag{1.1}$$

(μ, E are the moduli of elasticity for the binder and the fiber). The normal stress in the fiber and that tangential to the binder are computed from

$$\sigma_i = E\frac{\partial u_i}{\partial y}, \quad \tau_i = \mu\frac{u_{i+1} - u_i}{H}. \tag{1.2}$$

Komsomol'sk on the Amur. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 6, pp. 115-121, November-December, 1992. Original article submitted September 18, 1991.

